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A Model of Black Hole Evaporation and 4D Weyl Anomaly

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Received: 19 May 2017; Accepted: 12 June 2017; Published: 16 June 2017

Abstract: We analyze the time evolution of a spherically-symmetric collapsing matter from the point of view that black holes evaporate by nature. We consider conformal matters and solve the semi-classical Einstein equation $G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$ by using the four-dimensional Weyl anomaly with a large c coefficient. Here, $\langle T_{\mu\nu}\rangle$ contains the contribution from both the collapsing matter and Hawking radiation. The solution indicates that the collapsing matter forms a dense object and evaporates without horizon or singularity, and it has a surface, but looks like an ordinary black hole from the outside. Any object we recognize as a black hole should be such an object.

Keywords: black hole evaporation; self-consistent model; Weyl anomaly

1. Introduction and the Basic Idea

Black holes are formed by matters and evaporate eventually [1]. This process should be governed by the dynamics of a coupled quantum system of matter and gravity. It has been believed for a long time that taking the back reaction from the evaporation into consideration does not change the classical picture of black holes drastically. This is because evaporation occurs in the time scale $\sim a^3/l_p^2$ as a quantum effect, while collapse does in the time scale $\sim a$ as a classical effect[1]. Here, $a = 2GM$, and $l_p \equiv \sqrt{\hbar G}$. However, these two effects become comparable near the black hole. Recently, it has been discussed that the inclusion of the back reaction plays a crucial role in determining the time evolution of a collapsing matter [3–9].

We first explain our basic idea by considering the following process. Suppose that a spherically symmetric black hole with mass $M = \frac{a}{2G}$ is evaporating. Then, we consider what happens if we add a spherical thin shell to it. The important point here is that the shell will never go across "the horizon" because the black hole disappears before the shell reaches "the horizon".

To see this, we assume, for simplicity, that Hawking radiation goes to infinity without reflection and then describe the spacetime outside the black hole by the outgoing Vaidya metric [10]:

$$ds^2 = -\frac{r-a(u)}{r}du^2 - 2dudr + r^2 d\Omega^2, \tag{1}$$

where $M(u) = \frac{a(u)}{2G}$ is the Bondi mass. We assume that $a(u)$ satisfies:

$$\frac{da}{du} = -\frac{\sigma}{a^2}, \tag{2}$$

1 See, e.g., [2] for a classical analysis of collapsing matters.

where $\sigma = kNl_p^2$ is the intensity of the Hawking radiation. Here, N is the degrees of freedom of fields in the theory, and k is an $O(1)$ constant.

If the shell comes close to $a(u)$, the motion is governed by the equation for ingoing radial null geodesics:

$$\frac{dr(u)}{du} = -\frac{r(u) - a(u)}{2r(u)} \tag{3}$$

no matter what mass and angular momentum the particles constituting the shell have[2]. Here, $r(u)$ is the radial coordinate of the shell. This reflects the fact that any particle becomes ultra-relativistic near $r \sim a$ and behaves like a massless particle [11]. As we will show soon in the next section, we obtain the solution of (3):

$$\begin{aligned} r(u) &\approx a(u) - 2a(u)\frac{da}{du}(u) + Ce^{-\frac{u}{2a(u)}} \\ &= a(u) + \frac{2\sigma}{a(u)} + Ce^{-\frac{u}{2a(u)}} \longrightarrow a(u) + \frac{2\sigma}{a(u)} \,. \end{aligned} \tag{4}$$

This means the following (see Figure 1): The shell approaches the radius $a(u)$ in the time scale of $O(2a)$, but during this time, the radius $a(u)$ itself is slowly shrinking as (2). Therefore, $r(u)$ is always apart from $a(u)$ by $-2a\frac{da}{du}$. Thus, the shell never crosses the radius $a(u)$ as long as the black hole evaporates in a finite time, which keeps the (u, r) coordinates complete outside "the horizon", $r > a(u)$.

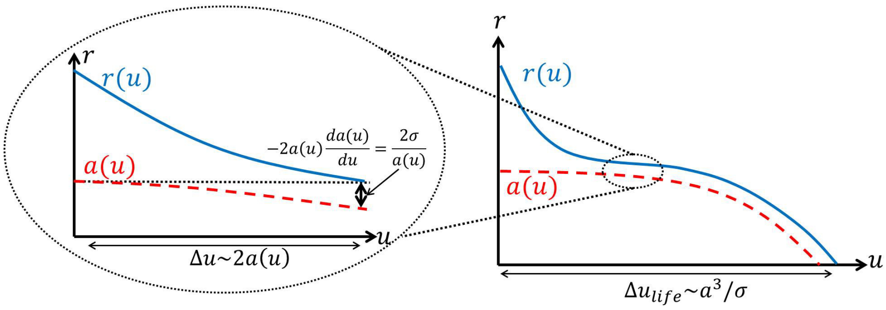

Figure 1. Motion of a shell or a particle near the evaporating black hole.

After the shell comes sufficiently close to $r = a + \frac{2\sigma}{a}$, the total system composed of the black hole and the shell behaves like an ordinary black hole with mass $M + \Delta M$, where ΔM is the mass of the shell. In fact, as we will see later, the radiation emitted from the total system agrees with that from a black hole with mass $M + \Delta M$.

We then consider a spherically symmetric collapsing matter with a continuous distribution and regard it as a set of concentric null shells. We can apply the above argument to each shell because its time evolution is not affected by the outside shells due to the spherical symmetry. Thus, we conclude that any object we recognize as a black hole actually consists of many shells. See Figure 2. Therefore, there is not a horizon, but a surface at $r = a + \frac{2\sigma}{a}$, which is a boundary inside which the matter is distributed[3]. If we see the system from the outside, it looks like an evaporating black hole in the ordinary picture. However, it has a well-defined internal structure in the whole region and evaporates like an ordinary object[4] [5].

[2] See Appendix I in [5] for a precise derivation.
[3] That is essential for particle creation is a time-dependent metric, but not the existence of horizons. A Planck-like distribution can be obtained even if there is no horizon [3,5,12].
[4] We keep using the term "black hole" even though the system is different from the conventional black hole that has a horizon.
[5] See also [13–17]. See, e.g., [18,19] for a black hole as a closed trapped region in the vacuum.

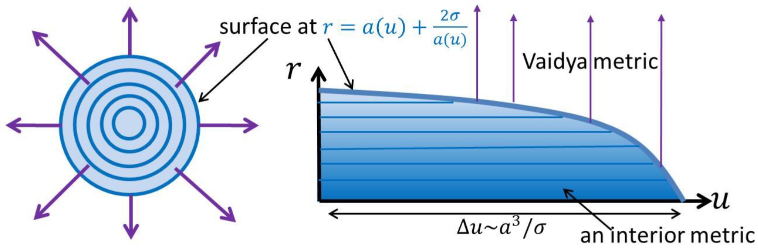

Figure 2. A black hole as an object that consists of many shells.

In order to prove this idea, we have to analyze the dynamics of the coupled quantum system of matter and gravity. As a first step, we consider the self-consistent equation:

$$G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle. \tag{5}$$

Here, we regard matter as quantum fields while we treat gravity as a classical metric $g_{\mu\nu}$. $\langle T_{\mu\nu} \rangle$ is the expectation value of the energy-momentum tensor operator with respect to the state $|\psi\rangle$ that stands for the time evolution of matter fields defined on the background $g_{\mu\nu}$. $\langle T_{\mu\nu} \rangle$ contains the contribution from both the collapsing matter and the Hawking radiation, and $|\psi\rangle$ is any state that represents a collapsing matter at $u = -\infty$.

In this paper, we consider conformal matters. Then, we show that $\langle T_{\mu\nu} \rangle$ on an arbitrary spherically symmetric metric $g_{\mu\nu}$ can be determined by the four-dimensional (4D) Weyl anomaly with some assumption and obtain the self-consistent solution of (5) that realizes the above idea. Furthermore, we can justify that the quantum fluctuation of gravity is small if the theory has a large c coefficient in the anomaly.

Our strategy to obtain the solution is as follows. We start with a rather artificial assumption that $\langle T^t{}_t \rangle + \langle T^r{}_r \rangle = 0$ (this is equivalent to $\langle T_{UV} \rangle = 0$ in Kruskal-like coordinates). By a simple model satisfying this assumption, we construct a candidate metric $g_{\mu\nu}$. We then evaluate $\langle T_{\mu\nu} \rangle$ on this background $g_{\mu\nu}$ by using the energy-momentum conservation and the 4D Weyl anomaly and show that the obtained $g_{\mu\nu}$ and $\langle T_{\mu\nu} \rangle$ satisfy (5). Next, we try to remove the assumption. We fix the ratio $\langle T^r{}_r \rangle / \langle T^t{}_t \rangle$, which seems reasonable for the conformal matter. Under this ansatz, the metric is determined from the trace part of (5), $G^\mu{}_\mu = 8\pi G \langle T^\mu{}_\mu \rangle$, where $\langle T^\mu{}_\mu \rangle$ is given by the 4D Weyl anomaly. On this metric, we calculate $\langle T_{\mu\nu} \rangle$ as before and check that (5) indeed holds.

This paper is organized as follows. In Section 2, we derive (4). In Section 3, we construct a candidate metric with the assumption $\langle T^t{}_t \rangle + \langle T^r{}_r \rangle = 0$. In Section 4, we evaluate $\langle T_{\mu\nu} \rangle$ on this metric and then check that (5) is satisfied. In Section 5, we remove the assumption and construct the general self-consistent solution. In Section 6, we rethink how the Hawking radiation is created in this picture.

2. Motion of a Thin Shell Near the Evaporating Black Hole

We start with the derivation of (4) [3–5]. That is, we solve (3) explicitly. Putting $r(u) = a(u) + \Delta r(u)$ in (3) and assuming $\Delta r(u) \ll a(u)$, we have:

$$\frac{d\Delta r(u)}{du} = -\frac{\Delta r(u)}{2a(u)} - \frac{da(u)}{du}. \tag{6}$$

The general solution of this equation is given by:

$$\Delta r(u) = C_0 e^{-\int_{u_0}^{u} du' \frac{1}{2a(u')}} + \int_{u_0}^{u} du' \left(-\frac{da}{du}(u') \right) e^{-\int_{u'}^{u} du'' \frac{1}{2a(u'')}},$$

where C_0 is an integration constant. Because $a(u)$ and $\frac{da(u)}{du}$ can be considered to be constant in the time scale of $O(a)$, the second term can be evaluated as:

$$\int_{u_0}^{u} du' \left(-\frac{da}{du}(u')\right) e^{-\int_{u'}^{u} du'' \frac{1}{2a(u'')}}$$
$$\approx -\frac{da}{du}(u) \int_{u_0}^{u} du' e^{-\frac{u-u'}{2a(u)}} = -2\frac{da}{du}(u)a(u)(1 - e^{-\frac{u-u_0}{2a(u)}}).$$

Therefore, we obtain:

$$\Delta r(u) \approx C_0 e^{-\frac{u-u_0}{2a(u)}} - 2\frac{da}{du}(u)a(u)(1 - e^{-\frac{u-u_0}{2a(u)}}),$$

which leads to (4):

$$r(u) \approx a(u) - 2a(u)\frac{da}{du}(u) + Ce^{-\frac{u}{2a(u)}}$$
$$= a(u) + \frac{2\sigma}{a(u)} + Ce^{-\frac{u}{2a(u)}} \longrightarrow a(u) + \frac{2\sigma}{a(u)}.$$

This result indicates that any particle gets close to:

$$R(a) \equiv a + \frac{2\sigma}{a} \tag{7}$$

in the time scale of $O(2a)$, but it will never cross the radius $a(u)$ as long as $a(u)$ keeps decreasing as (2)[6]. In the following, we call $R(a)$ the surface of the black hole.

Here, one might wonder if such a small radial difference $\Delta r = \frac{2\sigma}{a}$ makes sense, since it looks much smaller than l_p. However, the proper distance between the surface $R(a)$ and the radius a is estimated for the metric (1) as[7]:

$$\Delta l = \sqrt{\frac{R(a)}{R(a) - a}\frac{2\sigma}{a}} \approx \sqrt{2\sigma}\,. \tag{8}$$

In general, this is proportional to l_p, but it can be large if we consider a theory with many species of fields. In fact, in that case, we have:

$$\sigma \sim N l_p^2 \gg l_p^2. \tag{9}$$

We assume that N is large, but not infinite, for example, $O(100)$ as in the standard model. Then, $\Delta r = \frac{2\sigma}{a}$ is a non-trivial distance.

3. Constructing the Candidate Metric

The purpose of this section is to construct a candidate metric by considering a simple model corresponding to the process given in Section 1 [3,5]. At this stage, we do not mind whether it is a solution of (5) or not, which will be the task for the next section.

3.1. Single-Shell Model

As a preliminary for the next subsection, we begin with a simpler model [3]. See Figure 3.

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6 The above analysis is based on the classical motion of particles, but we can show that the result is valid even if we treat them quantum mechanically. See Section 2-B and Appendix A in [5].

7 For the general metric, the proper length in the radial direction is given by $\Delta l = \sqrt{g_{rr} - \frac{(g_{ur})^2}{g_{uu}}}\Delta r$. See [11].

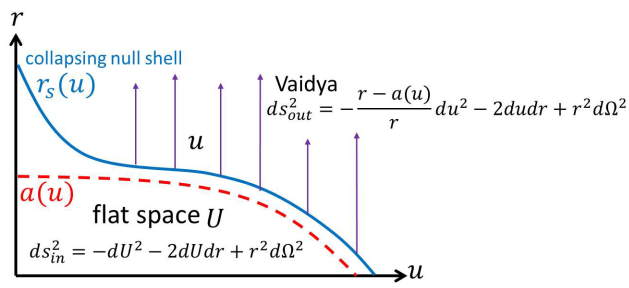

Figure 3. A spherical null shell evaporating in accordance with (2).

Suppose that a spherical null shell with mass $M = \frac{a}{2G}$ comes from infinity and evaporates like the ordinary black hole. Here, we consider the shell infinitely thin. We model this process by describing the spacetime outside the shell as the Vaidya metric (1) with (2). On the other hand, the spacetime inside it is flat because of spherical symmetry, and we express the metric by:

$$ds^2 = -dU^2 - 2dUdr + r^2\Omega^2. \quad (10)$$

Now, we have two time coordinates (u, U), and we need to connect them along the trajectory of the shell, $r = r_s(u)$. This can be done by noting that the shell is moving along an ingoing null geodesic in the metrics of the both sides, (1) and (10). Therefore, the junction condition is given by:

$$\frac{r_s(u) - a(u)}{r_s(u)}du = -2dr_s = dU. \quad (11)$$

This determines the relation between U and u for a given $a(u)$.

Generally, connecting two different metrics along a null hypersurface Σ leads to a surface energy-momentum tensor $T^{\mu\nu}_{\Sigma}$. Indeed, by using the Barrabes–Israel formalism [20,21], we can estimate the surface energy ϵ_{2d} and the surface pressure p_{2d} as[8]:

$$\epsilon_{2d} = \frac{a}{8\pi G r_s^2}, \quad p_{2d} = \frac{-\dot{a} r_s}{4\pi G(r_s - a)^2}. \quad (12)$$

Note that ϵ_{2d} is nothing but the energy per unit area of the shell with energy $M = \frac{a}{2G}$ and that the positive pressure p_{2d} is proportional to the energy being lost, $-\dot{a}(u) > 0$.

Thus, we have obtained the metric without coordinate-singularity that describes the formation and evaporation process of a black hole. Note again that we do not claim yet that this metric satisfies (5), but we here construct a candidate metric which formally expresses such a process.

3.2. Multi-Shell Model

Now, we consider a spherically-symmetric collapsing matter consisting of n spherical thin null shells. See Figure 4, where the position of the i-th shell is depicted by r_i.

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8 The surface tensor is given by $T^{\mu\nu}_{\Sigma} = (-\boldsymbol{k}\cdot\boldsymbol{v})^{-1}\delta(\tau)\,(\epsilon_{2d}k^{\mu}k^{\nu} + p_{2d}\sigma^{\mu\nu})$. Here, $\boldsymbol{v} = \frac{\partial}{\partial\tau}$ is the four-vector of a timelike observer with proper time τ who crosses the shell at $\tau = 0$, $\boldsymbol{k}$ is the ingoing radial null vector along the locus of the shell which is taken as $\boldsymbol{k} = \frac{2r_s(u)}{r_s(u)-a(u)}\partial_u - \partial_r$ for $r > r_s$ and $\boldsymbol{k} = 2\partial_U - \partial_r$ for $r < r_s$, and $\sigma^{\mu\nu}$ is the metric on the two-sphere ($\sigma_{\mu\nu}dx^{\mu}dx^{\nu} = r^2d\Omega^2$). See Appendix F in [5] for the detail.

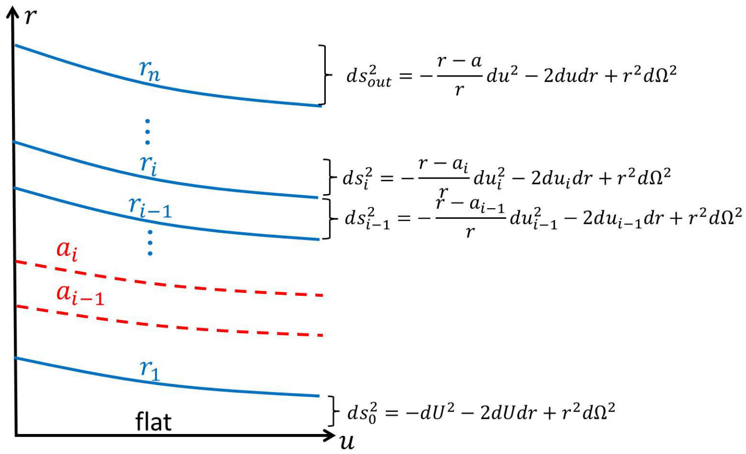

Figure 4. A multi-shell model.

We assume that each shell behaves like the ordinary evaporating black hole if we look at it from the outside. We postulate again that the radiation goes to infinity without reflection. Then, because of spherical symmetry, the region just outside the i-th shell can be described by the Vaidya metric:

$$ds_i^2 = -\frac{r-a_i(u_i)}{r}du_i^2 - 2du_idr + r^2d\Omega^2 \tag{13}$$

with:

$$\frac{da_i}{du_i} = -\frac{\sigma}{a_i^2} \tag{14}$$

for $i = 1\cdots n$. Here, $a_i = 2Gm_i \gg l_p$, and m_i is the energy inside the i-th shell (including the contribution from the shell itself). For $i = n$, $u_n = u$ is the time coordinate at infinity, and $a_n = a = 2GM$, where M is the Bondi mass for the whole system. On the other hand, the center, which is below the first shell, is the flat spacetime (10):

$$a_0 = 0, \quad u_0 = U. \tag{15}$$

In this case, the junction condition (11) is generalized to:

$$\frac{r_i - a_i}{r_i}du_i = -2dr_i = \frac{r_i - a_{i-1}}{r_i}du_{i-1} \quad \text{for } i = 1\cdots n. \tag{16}$$

This is equivalent to:

$$\frac{dr_i(u_i)}{du_i} = -\frac{r_i(u_i) - a_i(u_i)}{2r_i(u_i)} \tag{17}$$

and:

$$\frac{\mathrm{d}u_i}{\mathrm{d}u_{i-1}} = \frac{r_i - a_{i-1}}{r_i - a_i} = 1 + \frac{a_i - a_{i-1}}{r_i - a_i}. \tag{18}$$

As in the single-shell model, we have the surface energy-momentum tensor on each shell. By generalizing (12), we can show that the energy density $\epsilon_{2d}^{(i)}$ and the surface pressure $p_{2d}^{(i)}$ on the i-th shell are given by [5]:

$$\epsilon_{2d}^{(i)} = \frac{a_i - a_{i-1}}{8\pi G r_i^2}, \quad p_{2d}^{(i)} = -\frac{r_i}{4\pi G(r_i - a_i)^2}\left(\frac{da_i}{du_i} - \left(\frac{r_i - a_i}{r_i - a_{i-1}}\right)^2 \frac{da_{i-1}}{du_{i-1}}\right). \tag{19}$$

$\epsilon_{2d}^{(i)}$ expresses the energy density of the shell with energy $m_i = \frac{a_i - a_{i-1}}{2G}$. In the expression of $p_{2d}^{(i)}$, the first term corresponds to the total energy flux observed just above the shell, and the second one represents the energy flux below the shell that is redshifted due to the shell. Thus, the pressure is induced by the radiation from the shell itself[9].

3.3. The Candidate Metric

Finally, we take the continuum limit in the multi-shell model and construct the candidate metric [3–5]. Especially, we focus on a configuration in which each shell has already come close to $R(a_i)$:

$$r_i = R(a_i) = a_i + \frac{2\sigma}{a_i}, \tag{20}$$

where (7) has been used[10] (a more general case is discussed in [8]).

We first solve the Equation (16). By introducing:

$$\eta_i \equiv \log \frac{dU}{du_i}, \tag{21}$$

we have:

$$\begin{aligned}
\eta_i - \eta_{i-1} &= \log \frac{\frac{dU}{du_i}}{\frac{dU}{du_{i-1}}} = -\log \frac{du_i}{du_{i-1}} \\
&= -\log\left(1 + \frac{a_i - a_{i-1}}{r_i - a_i}\right) \\
&\approx -\frac{a_i - a_{i-1}}{r_i - a_i} = -\frac{a_i - a_{i-1}}{\frac{2\sigma}{a_i}} \\
&\approx -\frac{1}{4\sigma}\left(a_i^2 - a_{i-1}^2\right) .
\end{aligned} \tag{22}$$

Here, at the second line, we have used (18); at the third line, we have used (20) and assumed $\frac{a_i - a_{i-1}}{\frac{2\sigma}{a_i}} \ll 1$, which is satisfied for a continuous distribution; and at the last line, we have approximated $2a_i \approx a_i + a_{i-1}$. With the initial conditions (15), we obtain:

$$\eta_i = -\frac{1}{4\sigma} a_i^2. \tag{23}$$

Now, the metric at a spacetime point (U, r) inside the object is obtained by considering the shell that passes the point and evaluating the metric (13). We have at $r = r_i$:

$$\frac{r - a_i}{r} = \frac{r_i - a_i}{r_i} = \frac{\frac{2\sigma}{a_i}}{r_i} \approx \frac{2\sigma}{r^2} \tag{24}$$

$$\frac{du_i}{dU} = e^{-\eta_i} = e^{\frac{a_i^2}{4\sigma}} \approx e^{\frac{r^2}{4\sigma}}, \tag{25}$$

where (20) and (23) have been used. From these, we obtain the metric:

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9 See [5] for more detailed discussions.

10 Due to the spherical symmetry, the motion of each shell in the "local time" u_i is determined independently of the shells outside it. Therefore, the analysis for (7) can be applied to each shell.

$$\begin{aligned} ds^2 &= -\frac{r-a_i}{r}du_i^2 - 2du_i dr + r^2 d\Omega^2 \\ &= -\frac{r_i - a_i}{r_i}\left(\frac{du_i}{dU}\right)^2 dU^2 - 2\left(\frac{du_i}{dU}\right) dU dr + r^2 d\Omega^2 \\ &\approx -\frac{2\sigma}{r^2} e^{\frac{r^2}{2\sigma}} dU^2 - 2e^{\frac{r^2}{4\sigma}} dU dr + r^2 d\Omega^2. \end{aligned} \tag{26}$$

Note that this is static, although each shell is shrinking, and that it does not exist in the classical limit $\sigma \to 0$.

Thus, our candidate metric for the evaporating black hole is given by:

$$ds^2 = \begin{cases} -\frac{2\sigma}{r^2} e^{-\frac{R(a(u))^2 - r^2}{2\sigma}} du^2 - 2e^{-\frac{R(a(u))^2 - r^2}{4\sigma}} du dr + r^2 d\Omega^2, & \text{for } r \le R(a(u)), \\ -\frac{r-a(u)}{r} du^2 - 2dr du + r^2 d\Omega^2, & \text{for } r \ge R(a(u)), \end{cases} \tag{27}$$

which corresponds to Figure 2. Here, we have converted U to u by $du = e^{\frac{R(a(u))^2}{4\sigma}} dU$ and expressed (26) in terms of u. This metric is continuous at the surface $r = R(a(u)) = a(u) + \frac{2\sigma}{a(u)}$, where $a(u)$ decreases as (2).

Next, we consider a stationary black hole. Suppose that we put this object into the heat bath with temperature $T_H = \frac{\hbar}{4\pi a}$. Then, the ingoing energy flow from the bath and the outgoing one from the object become balanced each other[11], and the system reaches a stationary state, which corresponds to a stationary black hole in the heat bath [22] (see also Figure 5). The object has its surface at $r = R(a)$, where $a = \text{const}$. Then, the Vaidya metric for the outside spacetime is replaced with the Schwarzschild metric:

$$ds^2 = -\frac{r-a}{r}dt^2 + \frac{r}{r-a}dr^2 + r^2 d\Omega^2. \tag{28}$$

By introducing the time coordinate T around the origin as:

$$dT = dU + \frac{r^2}{2\sigma} e^{-\frac{r^2}{4\sigma}} dr, \tag{29}$$

we can write the interior metric (26) as:

$$ds^2 = -\frac{2\sigma}{r^2} e^{\frac{r^2}{2\sigma}} dT^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2. \tag{30}$$

Thus, by changing T to t through $dt = e^{\frac{R(a)^2}{4\sigma}} dT$, we obtain our candidate metric for the stationary black hole:

$$ds^2 = \begin{cases} -\frac{2\sigma}{r^2} e^{-\frac{R(a)^2 - r^2}{2\sigma}} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2, & \text{for } r \le R(a), \\ -\frac{r-a}{r} dt^2 + \frac{r}{r-a} dr^2 + r^2 d\Omega^2, & \text{for } r \ge R(a), \end{cases} \tag{31}$$

where $R(a) = a + \frac{2\sigma}{a}$ with $a = \text{const}$. The remarkable feature of (31) is that the redshift is exponentially large inside, and time is almost frozen in the region deeper than the surface by $\Delta r \gtrsim \frac{\sigma}{a}$.

4. Evaluating the Expectation Value of the Energy-Momentum Tensor

In this section, we evaluate the expectation value of the energy-momentum tensor $\langle T_{\mu\nu} \rangle$ in the candidate metrics (27) and (31) assuming that the matter is conformal. We show that $\langle T_{\mu\nu} \rangle$ can be determined by the four-dimensional Weyl anomaly and the energy-momentum conservation

[11] We can see how this "equilibration" occurs, by introducing interactions between radiations and matters. See Section 2-E in [5] for a detailed discussion.

$\nabla^\mu \langle T_{\mu\nu} \rangle = 0$ if we introduce a rather artificial assumption $\langle T_{UV} \rangle = 0$. Then, we show that the self-consist Equation (5) is indeed satisfied if σ in (27) and (31) is chosen properly.

4.1. Summary of the Assumptions So Far

We start with summarizing the assumptions that we have made to obtain the metric (27). Firstly, we assume that the system is spherically symmetric. Then, the time evolution of each shell is not affected by its exterior region after it becomes ultra-relativistic. Secondly, we assume that the radiation coming out of each shell flows to infinity without reflection. Then, the metric of each inter-shell region is given by the Vaidya metric.

We consider what these assumptions mean in terms of $\langle T_{\mu\nu} \rangle$. Here, we discuss in Kruskal-like coordinates (U, V): U and V are coordinates, such that outgoing and ingoing null lines are characterized by $U =$ const. and $V =$ const., respectively. Therefore, the second assumption means that in the inter-shell regions only $\langle T_{UU} \rangle$ is nonzero[12], and in particular,

$$\langle T_{UV} \rangle = 0. \tag{32}$$

Furthermore, noting the surface energy-momentum tensor (19), we find that $\epsilon_{2d}^{(i)}$ and $p_{2d}^{(i)}$ lead to nonzero values of $\langle T_{VV} \rangle$ and $\langle T^\theta{}_\theta \rangle = \langle T^\phi{}_\phi \rangle$, respectively, on each shell (see the footnote at (12)).

Thus, after taking the continuum limit, we have nonzero values for $\langle T_{\mu\nu} \rangle$ except for $\langle T_{UV} \rangle$. Therefore, the assumption we have made so far are essentially the spherical symmetry and (32). We keep the assumption (32) within this section and will remove it in the next section.

4.2. Relations among $\langle T_{\mu\nu} \rangle$ from the Energy-Momentum Conservation

We investigate the relations among the components of $\langle T_{\mu\nu} \rangle$ obtained from the energy-momentum conservation, which will be used to determine $\langle T_{\mu\nu} \rangle$. The general spherically symmetric metric can be expressed in Kruskal-like coordinates as:

$$ds^2 = -e^{\varphi(U,V)} dU dV + r(U,V)^2 d\Omega^2. \tag{33}$$

We assume that $\langle T_{\mu\nu} \rangle$ is spherically symmetric, that is, the non-zero components are:

$$\langle T_{UU} \rangle, \ \langle T_{VV} \rangle, \ \langle T_{UV} \rangle, \ \langle T^\theta{}_\theta \rangle = \langle T^\phi{}_\phi \rangle, \tag{34}$$

which depend only on U and V. Here, we keep $\langle T_{UV} \rangle$ for the convenience of the next section. Then, $\nabla^\mu \langle T_{\mu U} \rangle = 0$ and $\nabla^\mu \langle T_{\mu V} \rangle = 0$ are expressed as, respectively,

$$\langle T^\theta{}_\theta \rangle = -\frac{e^{-\varphi}}{r\partial_U r} \left[\partial_V (r^2 \langle T_{UU} \rangle) + \partial_U (r^2 \langle T_{UV} \rangle) - \partial_U \varphi (r^2 \langle T_{UV} \rangle) \right], \tag{35}$$

$$\langle T^\theta{}_\theta \rangle = -\frac{e^{-\varphi}}{r\partial_V r} \left[\partial_U (r^2 \langle T_{VV} \rangle) + \partial_V (r^2 \langle T_{UV} \rangle) - \partial_V \varphi (r^2 \langle T_{UV} \rangle) \right]. \tag{36}$$

The other components are satisfied trivially.

[12] We can see this explicitly as follows. Because the Vaidya metric has only G_{uu}, we can expect that only $\langle T_{uu} \rangle$ exists in the inter-shell regions. From the definitions of U and V, we have a transformation between (u, r) and (U, V) such that $\left(\frac{\partial u}{\partial V}\right)_U = 0$. Therefore, we evaluate $\langle T_{UU} \rangle = \left(\frac{\partial u}{\partial U}\right)^2 \langle T_{uu} \rangle \neq 0$, $\langle T_{UV} \rangle = \left(\frac{\partial u}{\partial U}\right)\left(\frac{\partial u}{\partial V}\right) \langle T_{uu} \rangle = 0$ and $\langle T_{VV} \rangle = \left(\frac{\partial u}{\partial V}\right)^2 \langle T_{uu} \rangle = 0$.

On the other hand, because the trace of the energy-momentum tensor is expressed as $\langle T^\mu{}_\mu\rangle = 2g^{UV}\langle T_{UV}\rangle + 2\langle T^\theta{}_\theta\rangle$, we have:

$$\langle T^\theta{}_\theta\rangle = \frac{1}{2}\langle T^\mu{}_\mu\rangle + 2e^{-\varphi}\langle T_{UV}\rangle. \tag{37}$$

Substituting (37) into (35) and (36), we obtain:

$$\partial_U(r^2\langle T_{UV}\rangle) - \left(\partial_U\varphi - \frac{2}{r}\partial_U r\right)(r^2\langle T_{UV}\rangle) = -\partial_V(r^2\langle T_{UU}\rangle) - \frac{1}{2}r\partial_U r e^{\varphi}\langle T^\mu{}_\mu\rangle, \tag{38}$$

$$\partial_V(r^2\langle T_{UV}\rangle) - \left(\partial_V\varphi - \frac{2}{r}\partial_V r\right)(r^2\langle T_{UV}\rangle) = -\partial_U(r^2\langle T_{VV}\rangle) - \frac{1}{2}r\partial_V r e^{\varphi}\langle T^\mu{}_\mu\rangle. \tag{39}$$

Once $\langle T^\mu{}_\mu\rangle$ is given, we can determine $\langle T_{\mu\nu}\rangle$ from these equations with some boundary conditions if one of the four functions (34) is known [23].

4.2.1. The Static Case

As a special case, we suppose that the spacetime is static. Then, $\varphi(U,V)$ and $r(U,V)$ satisfy:

$$\varphi(U,V) = \varphi(r(U,V)), \quad \partial_V r = -\partial_U r. \tag{40}$$

Then, we can rewrite (33) as:

$$ds^2 = -\frac{1}{B(r)}e^{A(r)}dT^2 + B(r)dr^2 + r^2 d\Omega^2, \tag{41}$$

where:

$$e^{\varphi(r)} = \frac{e^{A(r)}}{B(r)}, \quad \partial_V r = -\partial_U r = \frac{e^{\frac{A(r)}{2}}}{2B(r)} \tag{42}$$

and:

$$dU = dT - Be^{-\frac{A}{2}}dr, \quad dV = dT + Be^{-\frac{A}{2}}dr. \tag{43}$$

In this case, the expectation value of the energy-momentum tensor $\langle T_{\mu\nu}\rangle$ should also be static and satisfy:

$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(r)\rangle, \quad \langle T_{UU}\rangle = \langle T_{VV}\rangle. \tag{44}$$

Then, Equations (38) and (39) reduce to:

$$\partial_r(r^2\langle T_{UV}\rangle) - \left(\partial_r\varphi - \frac{2}{r}\right)(r^2\langle T_{UV}\rangle) = \partial_r(r^2\langle T_{UU}\rangle) - \frac{1}{2}re^{\varphi}\langle T^\mu{}_\mu\rangle. \tag{45}$$

4.3. *Evaluation of $\langle T_{\mu\nu}\rangle$ inside the Black Hole*

Now, we can evaluate $\langle T_{\mu\nu}\rangle$ in the metric (30) assuming (32) and (44). Here, we rewrite the metric (30) as (33) with (42) and:

$$A(r) = B(r) = \frac{r^2}{2\sigma}. \tag{46}$$

4.3.1. Boundary Conditions for $\langle T_{\mu\nu}\rangle$

We start with the boundary conditions. See Figure 5.

We first note that the region around $r = 0$ is kept to be a flat space. This is because the initial collapsing matter came from infinity with a dilute distribution. Then, the region inside the innermost shell in Figure 4 is flat due to the spherical symmetry, and it is almost frozen in time by the large redshift as in (27)[13]. Thus, the boundary conditions for $\langle T_{\mu\nu}\rangle$ are given by:

$$\langle T_{\mu\nu}\rangle|_{r\sim 0} = 0. \tag{47}$$

Note that this should be applied to both the evaporating and stationary black holes, because at any rate, black holes have been formed by collapse of matters.

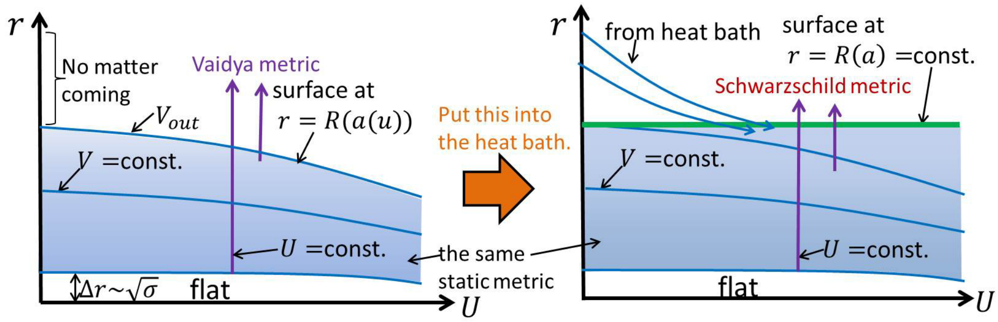

Figure 5. The boundary conditions. Left: The evaporating black hole in the vacuum. Right: The stationary black hole in the heat bath.

4.3.2. Employing $\nabla^\mu\langle T_{\mu\nu}\rangle = 0$

Now, we combine the energy-momentum conservation with the assumption (32). Under (32), (45) becomes:

$$\partial_r(r^2\langle T_{UU}\rangle) = \frac{1}{2} r e^{\varphi}\langle T^\mu{}_\mu\rangle. \tag{48}$$

Integrating this from zero to r for $\sqrt{\sigma} \ll r \le R(a)$, we have:

$$\begin{aligned} r^2\langle T_{UU}\rangle - (r^2\langle T_{UU}\rangle)|_{r=0} &= \frac{1}{2}\int_0^r dr' r' e^{\varphi(r')}\langle T^\mu{}_\mu(r')\rangle = \sigma\int_0^r dr' \frac{e^{\frac{r'^2}{2\sigma}}}{r'}\langle T^\mu{}_\mu(r')\rangle \\ &= \sigma e^{\frac{r^2}{2\sigma}}\int_0^r dr' \frac{e^{-\frac{r^2-r'^2}{2\sigma}}}{r'}\langle T^\mu{}_\mu(r')\rangle \\ &\approx \frac{\sigma}{r} e^{\frac{r^2}{2\sigma}}\langle T^\mu{}_\mu(r)\rangle\int_0^r dr' e^{-\frac{r}{\sigma}(r-r')} \\ &\approx \frac{\sigma^2}{r^2} e^{\frac{r^2}{2\sigma}}\langle T^\mu{}_\mu(r)\rangle. \end{aligned} \tag{49}$$

Here, at the first line, we have used (42) and (46); at the third line, we have assumed that $\langle T^\mu{}_\mu(r)\rangle$ does not change as rapidly as $e^{\frac{r^2}{2\sigma}}$, which will be checked soon, and used $e^{-\frac{1}{2\sigma}(r+r')(r-r')} \approx e^{-\frac{r}{\sigma}(r-r')}$, since the largest contribution comes from $r' \sim r$; at the final line, we have omitted the term proportional to $e^{-\frac{r^2}{\sigma}}$ for $r \gg \sqrt{\sigma}$. Finally, using the boundary condition (47), we have:

$$\langle T_{UU}\rangle = \langle T_{VV}\rangle = \frac{\sigma^2}{r^4} e^{\frac{r^2}{2\sigma}}\langle T^\mu{}_\mu\rangle. \tag{50}$$

[13] We will check the validity of (30) later. Indeed, (30) becomes almost flat at $r \sim \sqrt{\sigma}$, and can be connected to the flat spacetime.

On the other hand, under the assumption (32), (37) leads to:

$$\langle T^{\theta}{}_{\theta}\rangle = \frac{1}{2}\langle T^{\mu}{}_{\mu}\rangle. \tag{51}$$

Thus, all of the components of $\langle T_{\mu\nu}\rangle$ are determined by $\langle T^{\mu}{}_{\mu}\rangle$.

4.3.3. $\langle T^{\mu}{}_{\mu}\rangle$ from the 4D Weyl Anomaly

In the case of conformal matters, $\langle T^{\mu}{}_{\mu}\rangle$ is provided by the 4D Weyl anomaly once the metric is given [23–26]:

$$\langle T^{\mu}{}_{\mu}\rangle = \hbar c_{\mathrm{W}}\mathcal{F} - \hbar a_{\mathrm{W}}\mathcal{G}, \tag{52}$$

where $\mathcal{F} \equiv C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}$ and $\mathcal{G} \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$[14]. For the metric (30), $\mathcal{F}$ and $\mathcal{G}$ are calculated as:

$$\begin{aligned}\mathcal{F} &= \frac{A'^4}{12B^2} + \cdots = \frac{1}{3\sigma^2} + O\left(\frac{1}{\sigma r^2}\right)\\ \mathcal{G} &= -\frac{2A'^2}{r^2 B} + \cdots = O\left(\frac{1}{\sigma r^2}\right).\end{aligned} \tag{53}$$

Therefore, only the c-coefficient remains for $r \gg \sqrt{\sigma}$, and we obtain:

$$\langle T^{\mu}{}_{\mu}\rangle = \frac{\hbar c_{\mathrm{W}}}{3\sigma^2}, \tag{54}$$

which is constant and consistent with the assumption made in (49).

Thus, (50) and (51) are fixed as, respectively,

$$\langle T_{UU}\rangle = \langle T_{VV}\rangle = \frac{\hbar c_{\mathrm{W}}}{3r^4}e^{\frac{r^2}{2\sigma}}, \tag{55}$$

and:

$$\langle T^{\theta}{}_{\theta}\rangle = \frac{\hbar c_{\mathrm{W}}}{6\sigma^2}, \tag{56}$$

which means that the 4D Weyl anomaly provides the angular pressure [4,5][15].

4.4. The Self-Consistent Equation

Now, we can obtain the condition that the self-consistent Equation (5) holds, as follows. From (32), (55) and (56), we have:

$$-\langle T^{T}{}_{T}\rangle = \langle T^{r}{}_{r}\rangle = \frac{\hbar c_{\mathrm{W}}}{3\sigma}\frac{1}{r^2}, \quad \langle T^{\theta}{}_{\theta}\rangle = \frac{\hbar c_{\mathrm{W}}}{6\sigma^2}, \tag{57}$$

where we have used (43). On the other hand, the Einstein tensor for the metric (30) is calculated as:

$$-G^{T}{}_{T} = G^{r}{}_{r} = \frac{1}{r^2}, \quad G^{\theta}{}_{\theta} = \frac{1}{2\sigma}. \tag{58}$$

[14] We assume that the coefficients of the higher-curvature terms in the effective action are renormalized to order one. However,c_{W} and a_{W} are proportional to the degrees of freedom N because they are not canceled by counterterms [25]. Therefore, we can ignore the contributions from the higher curvature terms if $N \gg 1$.

[15] See, e.g., [27] for another application of the 4D Weyl anomaly to black holes.

Comparing (57) and (58), we conclude that (5) is satisfied if we identify:

$$\sigma = \frac{8\pi l_p^2 c_W}{3}. \tag{59}$$

We note that the dominant energy condition [21] is violated, $-\langle T^T{}_T\rangle \ll \langle T^\theta{}_\theta\rangle$, and that the interior is not a fluid in the sense $\langle T^r{}_r\rangle \ll \langle T^\theta{}_\theta\rangle$ [3–5].

We can check the validity of the classical gravity in (5). Indeed, in the macroscopic region $(r > l_p)$, all of the invariants for (30) are of order $\sim \frac{1}{\sigma}$:

$$R,\ \sqrt{R_{\mu\nu}R^{\mu\nu}},\ \sqrt{R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}} \sim \frac{1}{\sigma} \sim \frac{1}{l_p^2 c_W}. \tag{60}$$

They are smaller than the Planck scale if:

$$c_W \gg 1 \tag{61}$$

is satisfied. Therefore, macroscopic black holes $(a \gg l_p)$ can be described by the ordinary field theory. We do not need to consider quantum gravity except for the very small region $(r \sim l_p)$ or the last moment of the evaporation. (30) can be trusted for $r \gtrsim \sqrt{\sigma}$.

4.5. *Evaluation of $\langle T_{\mu\nu}\rangle$ outside the Black Hole*

In this subsection, we investigate $\langle T_{\mu\nu}\rangle$ in the outside region, $r > R(a)$, for both the evaporating and the stationary black holes.

4.5.1. The Evaporating Black Hole

First, we consider the evaporating back hole (27). Although we do not assume the static condition (44), we use a similar argument to the previous subsection. We first identify the boundary conditions. In the left of Figure 5, no ingoing matter comes after the collapsing matter at $U = -\infty$. Therefore, the boundary condition for the ingoing energy $\langle T_{VV}\rangle$ is given by:

$$\langle T_{VV}\rangle|_{U=-\infty} = 0 \ \text{ for } \ V > V_{out}, \tag{62}$$

where V_{out} labels the outermost shell. On the other hand, as we have shown in (55), the outgoing energy at the surface $r = R(a(U))$ is given by:

$$\langle T_{UU}\rangle|_{V=V_{out}} = \frac{\hbar c_W}{3R(a(U))^4} \ \text{ for } \ U \geq U_0. \tag{63}$$

Here, we have identified U in (33) with u in (1) so that $A = \frac{r^2 - R(a)^2}{2\sigma}$ as in (27). U_0 characterizes the time at which the outermost shell gets sufficiently close to $R(a(U))$ and starts to emit the radiation.

Using these boundary conditions and the conservation laws (38) and (39) with the assumption (32), we obtain (see Appendix A for the derivation):

$$r^2\langle T_{UU}\rangle = \frac{\hbar c_W}{3R(a(U))^2} + \frac{1}{2}\int_{R(a(U)),U=\text{const.}}^{r(U,V)} dr(r - a(U))\langle T^\mu{}_\mu\rangle, \tag{64}$$

$$r^2\langle T_{VV}\rangle = -\int_{-\infty}^{U} dU' r(\partial_V r)^2\langle T^\mu{}_\mu\rangle. \tag{65}$$

Next, we evaluate $\langle T^\mu{}_\mu\rangle$ from (52). For the metric (27) for $r > R(a(u))$, we have $\mathcal{F} = \mathcal{G} = \frac{12a(U)^2}{r^6}$ and obtain:

$$\langle T^\mu{}_\mu\rangle = 12\hbar(c_W - a_W)\frac{a(U)^2}{r^6}, \tag{66}$$

which gives $\langle T^\theta{}_\theta\rangle$ through (51). From (64) and (66), we obtain:

$$r^2\langle T_{UU}\rangle \approx \hbar\left(\frac{c_W}{3}+\frac{3(c_W-a_W)}{10}\right)\frac{1}{a(u)^2}+6\hbar(c_W-a_W)a(u)^2\left(-\frac{1}{4r^4}+\frac{a(u)}{5r^5}\right), \quad (67)$$

where $R(a)\approx a$ has been used. On the other hand, (65) cannot be evaluated explicitly due to the time dependence of $a(U)$. Here, in order to estimate its order, we assume that $a(U)$ is approximately constant. Then, we can have (see Appendix A):

$$r^2\langle T_{VV}\rangle \sim \hbar(c_W-a_W)a^2\left(-\frac{1}{4r^4}+\frac{a}{5r^5}\right). \quad (68)$$

Note here that the anomaly leads to particle creation even outside the black hole. The sign of c_W-a_W depends on the kind of field [25]. For example, it is positive for a massless scalar field, and it is negative for a massless vector field[16]. When $c_W-a_W>0$, (67) indicates that the outgoing radiation increases by the amount $\frac{3\hbar(c_W-a_W)}{10}\frac{1}{a(U)^2}$ as it goes to infinity from the surface. On the other hand, from (68), we can see that the negative ingoing energy is created [23,25,28].

Now, we check the self-consistent Equation (5). First, from (66)–(68), we can see that $\langle T_{\mu\nu}\rangle\sim\frac{1}{a^4}$ at $r\sim a$, which represents the energy-momentum of the radiation around the black hole as in the Stefan–Boltzmann law $\sim T_H^4$. The amount of energy in the region around the black hole with the volume $V\sim a^3$ is estimated as $\langle T_{\mu\nu}\rangle V\sim\frac{1}{a}$, which is much smaller than the mass of the black hole itself, $M=\frac{a}{2G}$. In this sense, $\langle T_{\mu\nu}\rangle$ is negligible:

$$\langle T_{\mu\nu}\rangle \approx 0, \quad (69)$$

and the region outside the black hole is described by vacuum-like solutions, such as the Vaidya metric or the Schwarzschild metric.

We have seen so far that the metric (27) is the self-consistent solution describing the whole spacetime of the evaporating black hole. There is no horizon or singularity, but this object is the black hole in quantum mechanics (see Figure 6).

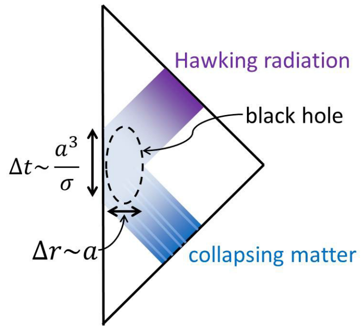

Figure 6. The Penrose diagram of the evaporating black hole described by the self-consistent solution (27).

4.5.2. The Stationary Black Hole

Next, we consider the stationary black hole in the heat bath (31). This time, we assume (44) in addition to (32) and use (48). We start with examining the boundary condition. See the right of

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16 However, $\frac{c_W}{3}+\frac{3(c_W-a_W)}{10}>0$ holds for any kind of massless fields [25], and $\langle T_{UU}\rangle$ is always positive at infinity. Here, the boundary condition (63) plays an important role. Later, we will discuss the origin of the radiation more closely.

Figure 5. Because the system is stationary, the surface is fixed at $r = R(a)$ =const., and there, the ingoing and outgoing energy flows are balanced as:

$$\langle T_{UU}\rangle|_{r=R(a)} = \langle T_{VV}\rangle|_{r=R(a)} = \frac{\hbar c_{\mathrm{W}}}{3R(a)^4}. \tag{70}$$

Here, we have used (55) and chosen the overall time scale as in (31), $A(r) = \frac{r^2-R(a)^2}{2\sigma}$.

Then, we calculate $\langle T^{\mu}{}_{\mu}\rangle$ from (51) and obtain the same value as (66) except for $a =$ const. We can evaluate $\langle T_{UU}\rangle$ from (48) with (70) and find that $\langle T_{UU}\rangle = \langle T_{VV}\rangle$ is given by (67) with $a =$ const.

Now, we study the self-consistent equation. Because we have the same order of $\langle T_{\mu\nu}\rangle$ as in the case of the evaporating black hole, we can follow the same reasoning for (69). That is, $\langle T_{\mu\nu}\rangle$ is negligible, and the metric outside the black hole is close to the Schwarzschild metric.

5. Generalization

We have assumed so far that the radiation emitted from each shell flows to infinity without reflection, which is expressed by (32). For a more realistic description, however, this assumption should be removed.

First, we discuss what $\langle T_{UV}\rangle \neq 0$ means. In the (U, V) coordinates (33), this is equivalent to the nonzero trace in the two-dimensional part (U, V):

$$\langle T^a{}_a\rangle \equiv \langle T^U{}_U\rangle + \langle T^V{}_V\rangle = 2g^{UV}\langle T_{UV}\rangle. \tag{71}$$

In a (t, r) coordinate system, in which the metric is diagonal, this is expressed as:

$$\langle T^a{}_a\rangle = \langle T^t{}_t\rangle + \langle T^r{}_r\rangle. \tag{72}$$

In other words, $\langle T_{UV}\rangle = 0$ is equivalent to $-\langle T^t{}_t\rangle = \langle T^r{}_r\rangle$, which is indeed satisfied by the previous self-consistent solution as in (57). Therefore, we characterize $\langle T_{UV}\rangle \neq 0$ by introducing a function $f(t, r)$ such that:

$$\frac{\langle T^r{}_r\rangle}{-\langle T^t{}_t\rangle} \equiv \frac{1-f}{1+f}. \tag{73}$$

$f = 0$ corresponds to $-\langle T^t{}_t\rangle = \langle T^r{}_r\rangle$. Here, if we require $\langle T^r{}_r\rangle \geq 0$ and $-\langle T^t{}_t\rangle > 0$, f must satisfy $|f| \leq 1$. In the following arguments, we assume that the matters are conformal.

5.1. Determination of the Interior Metric

For simplicity, we consider a stationary black hole in the heat bath. More precisely, we describe the exterior by the Schwarzschild metric (28), and parametrize the interior metric by (41) [4]. Then, we assume that $\langle T_{\mu\nu}\rangle$ is static and satisfies (44). Our program is to fix two functions $A(r)$ and $B(r)$ by two equations.

The first equation comes from (73). Once $f(r)$ is given, we rewrite the relation (73), by using the self-consistent Equation (5) for the ansatz (41), as:

$$\frac{2}{1+f} = \frac{G^r{}_r}{-G^t{}_t} + 1 = \frac{r\partial_r A}{B - 1 + r\partial_r \log B}. \tag{74}$$

In order to build the second equation, we apply the Weyl anomaly Equation (52) to the trace of (5):

$$G^{\mu}{}_{\mu} = 8\pi G\langle T^{\mu}{}_{\mu}\rangle = \gamma\mathcal{F} - \alpha\mathcal{G}, \tag{75}$$

where we have introduced the notations $\gamma \equiv 8\pi G\hbar c_{\mathrm{W}}$ and $\alpha \equiv 8\pi G\hbar a_{\mathrm{W}}$.

Here, we assume that for $r \gg l_p$, $A(r)$ and $B(r)$ are large quantities of the same order as expected from (46):

$$A(r) \sim B(r) \gg 1\,. \tag{76}$$

Then, the first Equation (74) becomes approximately:

$$A' = \frac{2B}{(1+f)r}, \tag{77}$$

where $A' = \partial_r A$, and we have used $B \gg 1, r\partial_r \log B$. Next, in order to examine what terms dominate in (75) for $r \gg l_p$, we replace A, B and r with μA, μB and $\sqrt{\mu}r$, respectively, and pick up the terms with the highest powers of μ. Then, we have:

$$\frac{A'^2}{2B} + \cdots = \gamma\left(\frac{A'^4}{12B^2} + \cdots\right) - \alpha\left(-\mu^{-1}\frac{2A'^2}{r^2 B} + \cdots\right). \tag{78}$$

Therefore, in the leading order of r, (75) becomes $\frac{A'^2}{2B} = \gamma\frac{A'^4}{12B^2}$, that is,

$$B = \frac{\gamma}{6}A'^2\,. \tag{79}$$

It is natural to expect that the dimensionless function $f(r)$ is a constant for conformal fields [4]:

$$f(r) = \text{const.} \tag{80}$$

Then, from (77)–(80), we obtain:

$$A = \frac{r^2}{2(1+f)\sigma_f}, \quad B = \frac{r^2}{2\sigma_f}, \tag{81}$$

where we have defined:

$$\sigma_f \equiv \frac{8\pi l_p^2 c_W}{3(1+f)^2}. \tag{82}$$

Thus, the interior metric is determined as:

$$ds^2 = -\frac{2\sigma_f}{r^2}e^{\frac{r^2}{2(1+f)\sigma_f}}dT^2 + \frac{r^2}{2\sigma_f}dr^2 + r^2 d\Omega^2. \tag{83}$$

Indeed, this is a generalization of (30) because (82) and (83) become (59) and (30), respectively, if we set $f = 0$. Redefining the overall scale of time and connecting the metric with the Schwarzschild metric, we reach the generalized metric for the stationary black hole:

$$ds^2 = \begin{cases} -\frac{2\sigma_f}{r^2}e^{-\frac{R(a)^2 - r^2}{2(1+f)\sigma_f}}dt^2 + \frac{r^2}{2\sigma_f}dr^2 + r^2 d\Omega^2, & \text{for } r \le R(a)\,, \\ -\frac{r-a}{r}dt^2 + \frac{r}{r-a}dr^2 + r^2 d\Omega^2, & \text{for } r \ge R(a)\,, \end{cases} \tag{84}$$

where $R(a) = a + \frac{2\sigma_f}{a}$. The metric for the evaporating one is obtained with the outside metric replaced by the Vaidya metric (1).

5.2. Check of the Self-Consistent Equation

As in Section 4, we now evaluate $\langle T_{\mu\nu}\rangle$ in the metric (84) and check the self-consistent equation. Because we assume that $\langle T_{\mu\nu}\rangle$ is static, we have to determine three functions of r: $\langle T_{UU}\rangle = \langle T_{VV}\rangle$, $\langle T_{UV}\rangle$ and $\langle T^{\theta}{}_{\theta}\rangle$.

5.2.1. Evaluation of $\langle T_{\mu\nu}\rangle$ inside the Black Hole

First we determine $\langle T_{\mu\nu}\rangle$ in the interior metric (83), which can be expressed by (33) with (81). We assume (80) and express the relation (73) as:

$$\langle T_{UV}\rangle = f\langle T_{UU}\rangle, \tag{85}$$

where we have used (43). Thus, only $\langle T_{UU}\rangle$ and $\langle T^{\theta}{}_{\theta}\rangle$ are left as unknown functions.

We then substitute (85) into (45) and obtain:

$$\partial_r((f-1)r^2\langle T_{UU}\rangle) - f\left(\partial_r\varphi - \frac{2}{r}\right)(r^2\langle T_{UU}\rangle) = -\frac{1}{2}re^{\varphi}\langle T^{\mu}{}_{\mu}\rangle. \tag{86}$$

Using (80), (81) and $\partial_r\varphi \approx \partial_r A = \frac{r}{(1+f)\sigma_f} \gg \frac{2}{r}$ for $r \gg l_p$, we reach:

$$\partial_r(r^2\langle T_{UU}\rangle) + \frac{f}{(1-f^2)\sigma_f}r(r^2\langle T_{UU}\rangle) = \frac{\sigma_f}{(1-f)r}e^{\frac{r^2}{2(1+f)\sigma_f}}\langle T^{\mu}{}_{\mu}\rangle. \tag{87}$$

The solution can be expressed as:

$$r^2\langle T_{UU}(r)\rangle = C(r)e^{-\frac{f}{2(1-f^2)\sigma_f}r^2}, \tag{88}$$

where $C(r)$ satisfies

$$\partial_r C = \frac{\sigma_f}{(1-f)r}e^{\frac{r^2}{2(1-f^2)\sigma_f}}\langle T^{\mu}{}_{\mu}\rangle. \tag{89}$$

This equation can be solved easily as:

$$\begin{aligned} C(r) - C(0) &= \frac{\sigma_f}{(1-f)}\int_0^r dr'\frac{1}{r'}e^{\frac{r'^2}{2(1-f^2)\sigma_f}}\langle T^{\mu}{}_{\mu}(r')\rangle \\ &\approx \frac{(1+f)\sigma_f^2}{r^2}e^{\frac{r^2}{2(1-f^2)\sigma_f}}\langle T^{\mu}{}_{\mu}(r)\rangle, \end{aligned} \tag{90}$$

where we have employed almost the same technique as in (49). Here, the boundary condition (47) means $C(0) = 0$. Then, we reach:

$$r^2\langle T_{UU}(r)\rangle = \frac{(1+f)\sigma_f^2}{r^2}e^{\frac{r^2}{2(1+f)\sigma_f}}\langle T^{\mu}{}_{\mu}(r)\rangle. \tag{91}$$

Applying the Weyl anomaly formula (52) to the metric (83) and using the same estimation as (53), we have:

$$\langle T^{\mu}{}_{\mu}\rangle = \frac{\hbar c_W}{3(1+f)^4\sigma_f^2} = \frac{3}{(8\pi)^2 G l_p^2 c_W}, \tag{92}$$

where at the second equality, we have used (82)[17]. Substituting this into (91), we obtain:

$$r^2\langle T_{UU}(r)\rangle = \frac{\hbar c_W}{3(1+f)^3 r^2}e^{\frac{r^2}{2(1+f)\sigma_f}}, \tag{93}$$

[17] We note that $\langle T^{\mu}{}_{\mu}\rangle$ is independent of f.

which reduces to (55) if $f = 0$. Then, from (37), (85) and (93), we obtain:

$$\langle T^\theta{}_\theta \rangle = \frac{3}{2(8\pi)^2 G l_p^2 c_W} + \frac{f}{8\pi G(1+f)r^2} \approx \frac{3}{2(8\pi)^2 G l_p^2 c_W}. \tag{94}$$

Now, we can check the self-consistent Equation (5) explicitly. Using (85), (93) and (43), we have:

$$-\langle T^t{}_t \rangle = \frac{\hbar c_W}{3\sigma_f (1+f)^2 r^2} = \frac{1}{8\pi G r^2} \tag{95}$$

$$\langle T^r{}_r \rangle = \frac{\hbar c_W (1-f)}{3\sigma_f (1+f)^3 r^2} = \frac{1}{8\pi G r^2}\frac{1-f}{1+f} \tag{96}$$

where at the second equality, we have used (82). On the other hand, we have for the metric (83):

$$-G^t{}_t = \frac{1}{r^2}, \quad G^r{}_r = \frac{1}{r^2}\frac{1-f}{1+f}, \quad G^\theta{}_\theta = \frac{1}{2(1+f)^2 \sigma_f} = \frac{3}{16\pi l_p^2 c_W}. \tag{97}$$

Comparing (94), (95) and (96) with (97), we find that (5) is indeed satisfied.

Finally, we see that the quantum fluctuation of gravity is small also in the general case. In fact, the invariants of (83) are given by:

$$R, \ \sqrt{R_{\mu\nu}R^{\mu\nu}}, \ \sqrt{R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}} \sim \frac{1}{(1+f)^2\sigma_f} \sim \frac{1}{l_p^2 c_W}, \tag{98}$$

where (82) has been used. They are small compared with the Planck scale, and therefore, the fluctuation is small if (61) is satisfied.

5.2.2. Evaluation of $\langle T_{\mu\nu} \rangle$ outside the Black Hole

Next, we consider the outside region, $r > R(a)$, of the metric (84). As we have seen in the previous section, $\langle T_{\mu\nu} \rangle$ outside the black hole is so small that the modification from the Schwarzschild or Vaidya metric is negligible, although the precise condition to fix $\langle T_{\mu\nu} \rangle$ is not known. In this subsection, as a simple example, we fix $\langle T_{UU} \rangle$ by hand and determine $\langle T_{UV} \rangle$. Then, we show that the region outside the black hole can be described approximately by the Schwarzschild metric.

We assume:

$$\langle T_{UU}(r) \rangle = \frac{\hbar c_W}{3(1+f)^3 R(a)^2}\frac{1}{r^2}, \tag{99}$$

where f is a constant given by (80). This means that the total flux emitted from the surface at $r = R(a)$ is kept outside (see (93) for $A = \frac{r^2 - R(a)^2}{2(1+f)\sigma_f}$), while the other effects (such as particle creation outside the black hole by the anomaly in Subsection 4.5) do not contribute to $\langle T_{UU} \rangle$. Furthermore, we take for simplicity:

$$\langle T_{UV} \rangle|_{r=R(a)} = 0, \tag{100}$$

as the boundary condition. We note that (99) and (100) are not given by some principle, but chosen by hand as an example.

Then, the first term in the right-hand side of (45) vanishes, while the second term is given through the Weyl anomaly by (66) with a =const. Solving (45) with the method of variation of constants under (100), we obtain[18]:

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[18] For given $\langle T_{UU} \rangle$ and $\langle T^\mu{}_\mu \rangle$, we solve (45) with respect to $r^2\langle T_{UV} \rangle$ and have $r^2\langle T_{UV}(r) \rangle = D(r)e^{\varphi - 2\log r} = D(r)\frac{r-a}{r^3}$, where $e^\varphi = \frac{r-a}{r}$ has been used. Then, $D(r)$ satisfies $\partial_r D = \frac{r^3}{r-a}\partial_r(r^2\langle T_{UU}(r)\rangle) - \frac{1}{2}r^3\langle T^\mu{}_\mu \rangle$. Applying (99) and (66) to this and integrating it from $R(a)$ to r, we obtain (101) if (100) is considered.

$$\langle T_{UV}(r)\rangle = 3\hbar(c_W - a_W)a^2\left(\frac{1}{r^2} - \frac{1}{R(a)^2}\right)\frac{r-a}{r^5}. \tag{101}$$

This behaves $\sim \frac{a^2}{r^4}$ for $r \gg a$, which decreases faster than (99) and does not contribute to the flux at infinity. Using (66) and (101), we can evaluate $\langle T^\theta{}_\theta\rangle$ through (37) as:

$$\langle T^\theta{}_\theta\rangle = 6\hbar(c_W - a_W)\frac{a^2}{r^4}\left(\frac{2}{r^2} - \frac{1}{R(a)^2}\right). \tag{102}$$

Thus, $\langle T_{\mu\nu}\rangle \sim \frac{1}{a^4}$ around $r \sim a$, and we can regard $\langle T_{\mu\nu}\rangle \approx 0$ by the same reasoning for (69). Therefore, (5) is satisfied by (84).

6. Hawking Radiation

In this section, we discuss how close the object that we are considering is to the black hole in the conventional picture.

6.1. Amount of the Radiation

First we show that the object emits the same amount of radiation as the conventional black hole. We prove that the energy flux at r is given by:

$$J(r) = \frac{4\pi\hbar c_W}{3(1+f)^2 r^2} = \frac{\sigma_f}{2Gr^2}, \tag{103}$$

where J is the energy passing through the ingoing spherical null surface at r per unit time. Here, the time is "the local time at r" such as u_i in (13) for the multi-shell model (then, (103) agrees with the right-hand side of (14)). More precisely, we define J by[19]:

$$J(r) \equiv 4\pi r^2 e^{-A}(\langle T_{UU}\rangle + \langle T_{UV}\rangle). \tag{104}$$

We can easily show that (104) becomes (103) by using (81), (85) and (93). Note that (103) means that the c-coefficient determines the intensity of the Hawking radiation, and the effect of f is to decrease the flux [4,5].

Now, we apply (103) to the surface $r = R(a)$ and obtain the energy flux emitted by the object:

$$J(R(a)) = \frac{\sigma_f}{2GR(a)^2}, \tag{105}$$

which agrees with the amount of the radiation emitted by the black hole in the conventional picture.

Here, we point out that we can obtain the energy spectrum of the radiation by solving the wave equation in the metric (27) under the Eikonal approximation. Indeed, it turns out to be the Planck-like distribution with the Hawking temperature [3,5].

6.2. Insensitivity to the Detail of the Initial Wave Function

Next, we argue that the expectation value of the energy momentum tensor is determined by the overall geometry and does not depend on the detail of the initial wave function. To see this, we start

---

[19] We can see that this definition is consistent with the concept of J, as follows. To do that, we first note that (14) suggests u_i as the natural time for description of the evaporation of each shell, and that in the continuum limit the redshift factor between U and u_i is $e^{\frac{A}{2}}$, as (25) shows. Then, we introduce the energy-momentum vector observed by $\boldsymbol{u}$ as $P^\mu \equiv -\langle T^\mu{}_\nu\rangle u^\nu$. Here, $\boldsymbol{u}$ is the four-vector with time u_i, which is defined by $\boldsymbol{u} \equiv e^{-\frac{A}{2}}\left(\frac{\partial}{\partial U}\right)_r = e^{-\frac{A}{2}}\left[\left(\frac{\partial}{\partial U}\right)_V + \left(\frac{\partial}{\partial V}\right)_U\right]$. Here, we have used (29) and (43). Thus, we can identify J with $J = 4\pi r^2(-P^\mu k_\mu)$, where $\boldsymbol{k} \equiv e^{-\frac{A}{2}}\left(\frac{\partial}{\partial U}\right)_V$ is the ingoing null vector along the shell.

with reexamining the analysis (49) of $\nabla^\mu\langle T_{\mu U}\rangle = 0$. If we integrate it from $r = r_0$ instead of $r = 0$, we have:

$$r^2\langle T_{UU}\rangle = (r^2\langle T_{UU}\rangle)|_{r_0} + \frac{\sigma^2}{r^2}\langle T^\mu{}_\mu(r)\rangle e^{\frac{r^2}{2\sigma}}(1 - e^{-\frac{r}{\sigma}(r-r_0)}). \tag{106}$$

Here, the last term vanishes for such r_0 that $\frac{r}{\sigma}(r - r_0) \gg 1$, and the first term is negligible, unless it is as large as $O(r^{-2}e^{\frac{r^2}{2\sigma}})$. Thus, even if we do not use the boundary condition (47), we obtain the same result (50).

This indicates that the amount of the radiation is determined universally by the geometry. Indeed, as is shown in (50), $\langle T_{UU}\rangle$ is produced at each point in the interior through the 4D Weyl anomaly (52), which is independent of the state, but is determined by the metric (30). Furthermore, while we have assumed the configuration (20) to obtain the metric (30), it has been shown by [8] that (30) is asymptotically reached from any initial distribution of mass and velocity of the matter. In this sense, the radiation occurs universally in collapsing processes, whose amount is given by (105).

Here, we emphasize that the 4D Weyl anomaly plays a crucial role in our picture of black holes. As (51) shows, the anomaly induces the strong angular pressure (56) [28–32]. It is so strong in the metric (30) that the object can be stable against the strong gravitational force[20] [21].

6.3. Fate of the Incoming Matter

Finally, we discuss the information problem. In our picture, the matter fields simply propagate in the background metric as in the ordinary quantum field theory on curved spacetime, and nothing special happens during the time evolution. Therefore, it is natural to expect that the collapsing matter itself eventually comes back as the radiation[22].

Indeed, we can get a clue to this by a simple analysis [5]. Suppose that a particle with energy $\sim \frac{\hbar}{a}$ comes close to the black hole and becomes a part of it. Then, it starts to emit radiation. As the particle loses energy, its wavelength increases. If the wavelength gets larger than the size of the black hole, then the particle can no longer stay in it. We can estimate the time scale of this process as $\sim a \log \frac{a}{\sqrt{\sigma}}$, which is much shorter than that of the evaporation $\sim \frac{a^3}{\sigma}$.

Therefore, one of the important future works is to solve the wave equation in the self-consistent metric (27) more precisely[23]. If we succeed in it, we should be able to understand how the information of the collapsing matter comes back and especially what happens to the baryon number conservation [5] [24].

7. Summary and Discussion

Our solution tells what the black hole is. The collapsing matter becomes a dense object and evaporates eventually without forming a horizon or singularity. It has a surface instead of the horizon, but looks like an ordinary black hole from the outside. In the interior, the non-trivial structure is formed, where the matter and the Hawking radiation can interact. This can provide a possible solution to the information problem.

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20 We can see explicitly this by constructing the Tolman–Oppenheimer–Volkoff equation with $\langle T^r{}_r\rangle \neq \langle T^\theta{}_\theta\rangle$ and using $-\langle T^t{}_t\rangle, \langle T^r{}_r\rangle \ll \langle T^\theta{}_\theta\rangle$.

21 See also [33].

22 The entropy can also be understood by the matter in the interior. The area law is reproduced by evaluating the entropy density and integrating it over the proper volume of the interior region. See Section 4-F and Appendix H in [5]. There are other approaches using the interior volume. See, e.g., [34].

23 See, e.g., [35,36] for the analysis of matter fields around the black hole.

24 There are many interesting approaches for the information problem. In [37–39], analyses are made based on an infalling observer, and in [40–42], the black hole is identified with a gravitational Bohr's hydrogen atom.

There remain problems to be clarified in the future. First, as we have mentioned, the important problem is to understand how the information comes back in this picture. To do it, we need to solve the wave equation in the self-consistent metric (27).

Second, although we have assumed a constant f to construct the metric (84), we do not understand its meaning yet. In principle, f should be determined by the dynamics of matters in the metric (84). Therefore, it is interesting to evaluate f concretely by considering a specific theory.

Third, the spherical symmetry has played an important role in our analysis. In the real world, however, we need to consider a rotating black hole, the outside of which is described by the Kerr metric. Although there is a conjecture on the interior metric for a slowly-rotating black hole [5], the general form is not known. It would be valuable if we can determine the interior metric by the 4D Weyl anomaly for the general case.

Fourth, we do not know yet how stable the metric (30) is for non-spherically symmetric perturbations. When investigating this problem, we need to be careful with the fact that the interior is not a fluid, as we have mentioned below (59).

Finally, astrophysics has entered into a new stage by the launch of gravitational wave detectors. For a new physics of black holes, it should be exciting to study an observable signal that exhibits some difference between the black holes in our picture and the conventional picture [43–45].

Acknowledgments: The authors thank the members of string theory group at National Taiwan University for valuable discussions. The present study was supported by KAKENHI 16H07445 and the RIKEN iTHES project. Y.Y. thanks the Department of Physics at Kyoto University for the hospitality.

Author Contributions: All the authors have substantially contributed to the research reported in this work and the writing of the manuscript. All have read and approved the final version.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Derivation of (64) and (65)

We derive (64) and (65). We first express the Vaidya metric (1) in the form of (33). We put $u = U$. Then, we introduce V as a label of an ingoing null line following (3): once an initial position for $r(U)$ in (3) is given, the solution is determined uniquely, which we denote by $\bar{r}(U,V)$. This plays roles of $r(U,V)$ in (33). Indeed, we have:

$$\begin{aligned} d\bar{r} &= \left(\frac{\partial \bar{r}}{\partial U}\right)_V dU + \left(\frac{\partial \bar{r}}{\partial V}\right)_U dV \\ &= -\frac{\bar{r}-a}{2\bar{r}} dU + \left(\frac{\partial \bar{r}}{\partial V}\right)_U dV, \end{aligned} \tag{A1}$$

replace dr in (1) with this, and obtain:

$$ds^2 = -2\left(\frac{\partial \bar{r}}{\partial V}\right)_U dUdV + \bar{r}(U,V)d\Omega^2, \tag{A2}$$

which means that $e^{\varphi(U,V)} = 2\left(\frac{\partial \bar{r}}{\partial V}\right)_U$.

Under (32), we integrate (38) from V_{out} to $V(> V_{out})$ along a fixed $U(\geq U_0)$:

$$
\begin{aligned}
(r^2\langle T_{UU}\rangle) - (r^2\langle T_{UU}\rangle)|_{V_{out}} &= -\frac{1}{2}\int_{V_{out}}^{V} dV' r\partial_U r e^{\varphi}\langle T^{\mu}{}_{\mu}\rangle \\
&= -\int_{V_{out}}^{V} dV' \left(\frac{\partial \bar{r}}{\partial V}\right)_U r\partial_U r\langle T^{\mu}{}_{\mu}\rangle \\
&= -\int_{r(U,V_{out}),U=\text{const.}}^{r(U,V)} dr r\partial_U r\langle T^{\mu}{}_{\mu}\rangle \\
&= \frac{1}{2}\int_{r(U,V_{out}),U=\text{const.}}^{r(U,V)} dr(r-a(U))\langle T^{\mu}{}_{\mu}\rangle.
\end{aligned}
\tag{A3}
$$

Here, at the second line, (A2) has been used; at the third line, we have used the fact that $dr = dV\left(\frac{\partial \bar{r}}{\partial V}\right)_U$ holds along a fixed U (see (A1)); at the last line, we employ (A1) again. Then, employing the boundary condition (63), we obtain (64).

Next, we derive (65). We integrate (39) with the assumption (32) and the boundary condition (62):

$$
\begin{aligned}
r^2\langle T_{VV}\rangle &= (r^2\langle T_{VV}\rangle)|_{U=-\infty} - \frac{1}{2}\int_{-\infty}^{U} dU' r\partial_V r e^{\varphi}\langle T^{\mu}{}_{\mu}\rangle \\
&= -\int_{-\infty}^{U} dU' r(\partial_V r)^2\langle T^{\mu}{}_{\mu}\rangle,
\end{aligned}
$$

where we have used $e^{\varphi(U,V)} = 2\left(\frac{\partial \bar{r}}{\partial V}\right)_U$ in (A2).

Then, we estimate its order assuming that $a(U)$ varies slowly, $a(U) \sim$ const. In this case, we can use (42) to have:

$$
\begin{aligned}
r^2\langle T_{VV}\rangle &= -\int_{\infty,V=\text{const.}}^{r} dr' \frac{1}{\partial_U r} r'(\partial_V r)^2\langle T^{\mu}{}_{\mu}\rangle \\
&= \int_{\infty,V=\text{const.}}^{r} dr' r'\partial_V r\langle T^{\mu}{}_{\mu}\rangle \\
&= \frac{1}{2}\int_{\infty,V=\text{const.}}^{r} dr'(r'-a)\langle T^{\mu}{}_{\mu}\rangle.
\end{aligned}
$$

Using (66), this becomes (68).

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